The Sound and Touch of Mathematics: a Prototype System

Frances L. Van Scoy, Takamitsu Kawai, Angela Fullmer, Kevin Stamper
West Virginia University
Iwona Wojciechowska - Alderson Broaddus College
Addis Perez, Jesir Vargas - University of Puerto Rico-Rio Piedras
Shelma Martinez - University of Puerto Rico-Mayaguez

The West Virginia Virtual Environments Laboratory is studying ways to use virtual environments technology to assist users with various disabilities. One problem of special interest to us is how to teach mathematics to students with vision disabilities. We are using the PHANToM and sonification techniques to display mathematical functions to visually impaired students.

Rationale

We agree with Moses and Cobb, "Part of the literacy standard, then, the floor for all students, must be this: when you leave middle school, you are ready to engage with the college preparatory sequence in high school. It's a moving target, but however it's defined, it must then be seen as another floor: when you leave high school, you must be able to engage college curricula in math and science, for full college credit." [1]

National Science Foundation Director Rita Colwell has said, "Every schoolchild must be educated for a productive and contributory place in an advanced information age... K through 12 is the real challenge. As a start, we begin with the assumption that all children can be educated in math and science. This may sound so elementary as to be downright silly! In some places, the educational approach is to sift and sort students early-on. This tells some students right at the starting gate that they can't master science and math -- that we do not expect them to succeed. This becomes a self-fulfilling prophecy, damning to the student and destructive for the country. We must believe in all children so that they learn to believe in themselves..." [2]

Our goal in this project is to help students with disabilities or with different learning styles learn pre-calculus so that college majors in science and engineering are available to them.

Previous Work

The current project brings together previous work in two areas, use of the PHANToM in displaying map information for pre-trip planning by those with visual impairments [3] and sonification of complex data sets [4-7]. Our sonification work emphasizes the development of algorithms for composing music from complex data sets which will be pleasant to listen to and will assist listeners in discovering data relationships which are not otherwise obvious.

In 2000 we developed a program for the PHANToM which accepts as input a mathematical function of one variable and then constructs a haptic model of a solid block on whose front face a groove representing the function has been carved. The user types on the computer keyboard to enter commands such as those for changing bounds of the domain and range, and the program uses spoken feedback to communicate with the user. A important component of our system is a compiler written using flex and bison which parses the user's function written in Fortran notation [8].

Stephen Brewster's group at the University of Glasgow has built a prototype multimodal haptic math system and done human factors research with both blind and sighted users. In the 2000 version of their system functions were provided by the human factors researchers and hard-coded.

They tested for effectiveness of different values of friction and representation of grid lines. They referenced their own work in sonification, writing, "An effective way of presenting the overview of the line graph will shorten the time required in this process and give blind users a better understanding about the graph. Using non-speech sound to provide this kind of quick overview [is] being investigated." [9]

Specification of New Prototypes

In 2001 we extended our prototype haptic math system to add:

(1) improvement of the user interface,

(2) display of functions of two variables, and

(3) sonification of functions of one variable

Improving User Interface

The existing version of haptic math provided only the basic functions needed to allow the user to enter a function and change some parameters. All input was done via the keyboard and made several assumptions to simplify the initial interface. Work this summer focused on identifying components required and/or useful in a more complete user interface. This includes not only the listing of these components but consideration as to how best to implement each of them to allow the greatest adaptability for users with varying disabilities or learning styles.

There are two main parts of the user interface: the informational display and the system controls.

The display part of the interface is concerned with how the function is represented to the user. This includes the graphic representation as drawn on the screen, the haptic representation as felt via the PHANToM, and the production of any auditory information. In regards to the multi-modal 'display' there were several issues considered. These included investigation into the way that the curve is drawn to provide a more uniform groove width at points with steep or zero slope, methods of informing the user (other than visually) of discontinuities in the graphed function and providing the user with alternative methods to explore the graph.

The system control part of the interface is concerned with how the user sets parameters, enters functions and in general conveys information to the system. For the system control part some issues considered include variations in the format of the functions that may be entered, the means of detecting and informing the user of errors in the function entered, the provision of user access to parameters and the ability of the user to selectively alter parts of the multi-modal display to adapt to individual needs. The existing version restricts the choices for system control keys to those on the left hand. This is an unnecessary and potentially problematic restriction for several reasons. It assumes a right-handed user. It assumes the traditional 'qwerty' keyboard, a problem since other keyboards are used by potential users. Also, it forces some command sequences to be rather arbitrary. The restriction is unnecessary as it is not possible on a qwerty keyboard to use only left hand sequences to enter characters such as the parentheses needed in the functions entered. This also forces the user to memorize the correspondence between keys and parameters - a potential problem for a novice user who may not even realize what parameters are available. It would also be desirable for the user to be able to alter the display. For example, a user might find the musical representation of the curve distracting and choose to turn it off.

Display of Functions of Two Variables

We also developed a new module for displaying functions of two variables.

First, the visual representation had to be changed. The original one-variable version carves a groove in a block of wood, giving a 3-d representation of a 2-d curve. In the case of a function of two variables, we are dealing with a 3-d surface. To obtain this surface, the plane (x,y) is triangulated and the z-value of the function is calculated for each triangulation vertex. This produces the set of 3-d points (x,y,x) that are used to create the tripoly mesh that PHANToM user can "feel." To make the surface easier to explore without falling off the surface, we added a bounding box.

The second change involved modifying the function parser to accommodate functions of two variables. This major modification required making changes to the interface between the display module and the parser. Additional arguments were needed. Also the type and meaning of one of the parameters was changed. The latter change was necessary, because of the precautions taken in the previous version against mathematical roundup errors.

Work remains to be done. Currently the function surface is displayed with the (x,y) plane horizontal. Additional display options can be added such as displaying the plane vertically or allowing the user to rotate the surface. Also for sighted users the color of the display should be changed and lighting added to create shadows. An artifact of the 2000 system is that the expression defining the function is re-parsed for every pair (x,y), with a major loss of efficiency. We need to restructure the system so that the function is parsed one time and then evaluated once for each (x,y) pair.

Sonification of Functions of One Variable

In sonifying functions of one variable we map a function to a line of music, mapping x-coordinates to time and y-coordinates to pitch. We do this in order to give the student a way of remembering the shape of a function in the belief that many students will be better able to remember a melody describing the sine function rather than only the muscular sensations of tracing it. We use two different encodings--one using the western chromatic scale and one using micropitch--in the belief that melodies based on the familiar scale will be easier to remember but that micropitch will give a more accurate representation of a function (since the chromatic scale encoding essentially represents continuous functions as step functions).

Figure 1 shows an example illustrating how we sonify functions of one variable. The music is that generated for f(x) = tan(x), for 0 = x = 2 p.

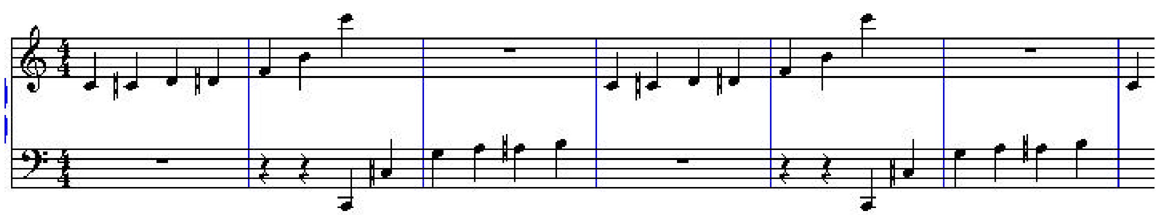

Figure 1. Sonification of Tangent Function

Continuing Work

We are now designing a human factors study for testing this PHANToM-based prototype with secondary school students and doing preliminary design for some course modules incorporating this approach.

References

[1] Moses, Robert P. and Charles E. Cobb, *Radical equations: math literacy and civil rights*, 2001, Boston: Beacon Press, p. 16.

[2] Colwell, Rita, address to DC Science Writers Association, September 8, 1998, available at http://www.nsf.gov/od/lpa/forum/colwell/rc80908.htm

[3] Van Scoy, Frances L. Vic Baker, Chaim Gingold, Eric Martino, Darrin Burton, "Mobility Training using a Haptic Interface: Initial Plans," in Salisbury, JK and Srinivasan, MA (Eds), "Proceedings of the Fourth PHANTOM Users Group Workshop," AI Lab Technical Report No. 1675 and RLE Technical Report No. 633, MIT, November 1999, available at http://www.sensable.com/haptics/community/pdf/PUG1999.PDF

[4] Van Scoy, Frances L., "Sonification of Complex Data Sets: An Example from Basketball," Proceedings of VSMM99 (Virtual Systems and MultiMedia), Dundee, Scotland, September 1-3, 1999.

[5] Van Scoy, Frances L., "Sonification of Remote Sensing Data: Initial Experiment," Information Visualisation 2000, London, UK, July 19-21, 2000

[6] Mutnuri, Sunitha, "Sonification of GIS Data," Master of Science in Computer Science problem report, Lane Department of Computer Science and Electrical Engineering, West Virginia University, December 2000.

[7] Wallace, Carl, "Sonification of Basketball Data," Master of Science in Computer Science problem report, Lane Department of Computer Science and Electrical Engineering, West Virginia University, December 2001.

[8] Van Scoy , Frances L., Takamitsu Kawai, Marjorie Darrah, Connie Darrah, "Haptic Display of Mathematical Functions for Teaching Students with Vision Disabilities: Design and Proof of Concept," In *Proceedings of the First Workshop on Haptic Human-Computer Interaction*, Glasgow, Scotland, August 31-September 1, 2000, available at http://www.dcs.gla.ac.uk/%7Estephen/workshops/haptic/papers/haptics_and_hci_workshop.zip

[9] Yu, W., R. Ramloll, and S. A. Brewster (2000). "Haptic graphs for blind computer user*s,"* in *Proceedings of the First Workshop on Haptic Human-Computer Interaction*, Glasgow, Scotland, August 31-September 1, 2000, available at http://www.dcs.gla.ac.uk/~stephen/papers/Assets2000.pdf

Acknowledgements

This work is being conducted in the West Virginia Virtual Environments Laboratory in the Lane Department of Computer Science and Electrical Engineering of West Virginia University and is supported by the EPSCoR programs of the National Science Foundation and the state of West Virginia and by the NSF CISE Research Experiences for Undergraduates program.

Darrin Burton of the WVU Center for Assistive Technology has provided valuable advice on usability of the system.

Nicole Johnson, a WVU McNair scholar, and Yusuke Abe, a Fairmont State College student, were active participants in VEL discussions about this project.